Reply to "Comment on 'Observation of matter wave beat phenomena in the macrodomain for electrons moving along a magnetic field' "

Ram K. Varma, A. M. Punithavelu, and S. B. Banerjee
Physical Research Laboratory, Ahmedabad 380 009, India
(Received 12 January 2004; published 30 August 2004)

The so-designated "complete classical explanation" by Unnikrishnan in the preceding Comment, of the macroscopic wave beats reported by us recently, is based on an assumed physical mechanism that has no experimentally established basis. Through this assumed physical process and the focusing properties of an electron beam in a magnetic field, he produces what are essentially "particle beats" as against the "wave beats" observed by us. Even assuming that such a physical process does exist, the consequences of his mechanism are shown here to be in serious contradiction with our experimental observations, thereby invalidating the claim that his model explains, in purely classical terms, our observations exhibiting the existence of macroscopic matter wave beats in the charged particle motion along a magnetic field.

DOI: 10.1103/PhysRevE.70.028502 PACS number(s): 41.75.−i, 41.85.−p, 41.90.+e, 03.65.Ta

I. INTRODUCTION

In the preceding Comment Unnikrishnan [1] has contested our interpretation of our recently reported observations [2] of the charged particle dynamics in a magnetic field. These observations are admittedly rather unusual and unexpected, as we have demonstrated that they signify the existence of hitherto unknown and unfamiliar wave mechanical manifestations which are revealed in a rather unexpected manner in the macrodomain of a few centimeters [2,3]. That is, the results exhibit the presence of matter wave interference effects along the direction of the magnetic field with a wavelength of a few centimeters (with the electron energy typically of a few hundred electron volts and a magnetic field of about 100 G).

It may be pointed out that these observations were, in fact, motivated following the prediction for such behavior by a theoretical formalism developed by Varma [4,5] which furnishes a set of probability-amplitude Schrödinger-like equations, being one dimensional along the magnetic field. A detailed account of the evolution and development of this paradigm represented by the set of Schrödinger-like equations has been presented in a recent review [6] where the experimental results referred to above [2] as well as other experimental results have also been reviewed.

The results reported in Ref. [2] are obviously rather enigmatic as we find that they are not amenable to explanation in terms of the classical Lorentz equation of motion to whose parameter domain they ostensibly belong. On the other hand, they do conform to the prediction of the theory of Refs. [4,5] whereby we have claimed that the results represent matter wave interference phenomena.

While it is eminently desirable to try to understand these enigmatic results in terms of classical Lorentz equation of motion, as has been attempted by Unnikrishnan, but as we shall show, in what follows, his attempt fails because the classical model that he has constructed is not only riddled with contradictions with some crucial aspects of our experimental results, but has also invoked a physical mechanism whose validity is far from being experimentally established. Moreover, much of his criticism and its tenor of our interpretation and identification in terms of the formalism of Refs. [4,5], which predicted these observed effects in the first place, seems to originate from a lack of understanding and appreciation of the nature of the formalism of Ref. [5]. It may therefore be appropriate to outline here briefly the basic elements of the formalism to emphasize its essentially wave mechanical origin so that its consequences, even if hitherto unknown and unfamiliar, are not considered external to known physical laws—quantum mechanics in the present case. Thus we clarify here that the formalism that we have based our interpretation on is not contrary to any known physical laws as Unnikrishnan erroneously implies in his comments.

II. BRIEF OUTLINE OF THE NATURE OF THE SCHRÖDINGER-LIKE MACRO-MATTER–WAVE FORMALISM

In the recent theoretical work by Varma [5] the wave functions governed by the Schrödinger-like equations, alluded to above, have been identified as quantum mechanical objects, being the transition amplitudes (matrix elements of the S matrix) induced by a perturbation from a very large Landau quantum number $N \gg 1$ (corresponding to the perpendicular degree of freedom of the charged particle in a magnetic field) to the neighboring ones $N \pm n (N \gg n \geqslant 1)$. It so turns out that these one-dimensional Schrödinger-like equations along the magnetic field have the large action $\mu = N\hbar$ in the role of $\hbar$. These transition amplitudes are themselves wave amplitudes and, because of the large value of the action $\mu = N\hbar$ (N being $\approx 10^8$ for a typical electron energy $\mathcal{E} = 1$ keV and a magnetic field $B = 100$ G), predict a matter wave behavior in the macroscopic domain of a few centimeters.

While the Schrödinger-like equations of Refs. [4,5] and reviewed in Ref. [6] are more general in their content, a simple and direct quantum mechanical derivation of the form of the transition amplitude from a large Landau quantum number N (around which the initial state of the ensemble of charged particle is centered) to the neighboring ones, $N\pm n$, is given in the Appendix to Ref. [2] and also reproduced in Sec. 7.3 of the review in [6]. If $\mathcal{E}_\perp$ represents the energy in the perpendicular degree of freedom of the particle, then $\mathcal{E}_\perp = N\hbar\Omega$ ($\Omega = eB/mc$ being the gyrofrequency), and $N = \mathcal{E}_\perp/\hbar\Omega$, as we noted earlier, turns out to be $\approx 10^8$ for an energy $\mathcal{E}_\perp \sim 1$ keV and magnetic field $B = 100$ G.

If $|N\rangle$ represents the Landau state corresponding to the quantum number N, then the transition (scattering) amplitude from the complete initial state $|N\rangle e^{ikx}$ to the final state $|N\pm n\rangle e^{ik'x}$, including the plane wave part along the field line, has been shown [2,6] to be given by

$$\langle N \pm n, k'|\tilde{H}|N,k\rangle' = \alpha e^{\pm in\Omega x/v_\parallel}, \quad \Omega = eB/mc, \tag{1}$$

where the prime on the matrix element indicates that the x coordinate is not integrated over; k' has been substituted in terms of k using total energy conservation involving the "perpendicular" and "parallel" energy, and $\tilde{H}$ denotes the perturbation Hamiltonian involving the perpendicular coordinate which induces the transition. This could be the perpendicular component of the electric field in the region of the electron gun or the grid wires with which the electron could strike while transiting from the gun to the plate.

The transition amplitude given by Eq. (1) has the form of a plane wave along the x direction, with a wave number $k_n = n\Omega/v_\parallel$, and which is clearly $\hbar$ independent. This transition amplitude is itself a wave amplitude, and it is this which describes macroscopic matter waves and would exhibit interference effects just like the de Broglie wave amplitude since, needless to repeat, it has a wave mechanical origin. Clearly, however, these waves are "derived" and not fundamental entities like the de Broglie waves and would exist only for partially bound systems—that is, those which are free in at least one of their degrees of freedom (the coordinate along the magnetic field in the present case). The resulting restriction, therefore, is that they are one dimensional along the magnetic field with their wave vector along the magnetic field.

Another important point to emphasize in relation to some points of the criticism proffered by Unnikrishnan is that these macroscopic matter waves represented by the transition amplitude (1) do not exist *a priori*, but are generated through scattering episodes involving, for example, the perpendicular component of the electric field in the region of the electron gun and/or the interaction with the wires of the grid through which the electron beam passes to reach the plate. Through these scattering episodes, the particles make a transition from quantum number N to $N\pm n$, with the total energy assumed to remain constant. The wavelength of these (macroscopic) matter waves, associated with the transition amplitudes, has the $\hbar$-independent expression $\lambda_n = 2\pi v_\parallel/n\Omega$ which is obviously of macroscopic dimension. It should be noted that we have the wavelength $\lambda_1 = 2\pi v_\parallel/\Omega$, but we also have the harmonics $\lambda_n = 2\pi v_\parallel/n\Omega$, corresponding to $n = 2, 3, \ldots$. With the parameter values $\mathcal{E} = 1$ keV and $B = 100$ g, one finds $\lambda_1 = 5$ cm.

What we claim to have demonstrated in Ref. [2] is the existence of one-dimensional interference effects involving the macroscopic matter wavelength $\lambda = 2\pi v_\parallel/\Omega$. With a given axial magnetic field B, a fixed distance L_p between the electron gun and the detector plate, and a grid close to the plate so that $(L_p - L_g) \ll L_p$ (L_g being the gun-grid distance) the wave algorithm with the above matter wavelength leads to a probability density at the plate given by Eq. (7.16) of Ref. [6] [or Eq. (10) of Ref. [2]]:

$$|\psi_p|^2 = \beta^2 + \gamma^2 + 2\alpha_0\beta[\beta^2 + 2\gamma^2] + [2\beta\gamma + 2\alpha_0\gamma(3\beta^2 + \gamma^2)]\cos kL_p + 2\alpha_0\beta\gamma^2 \cos 2kL_p, \tag{2}$$

with $k = \Omega/v_\parallel$, where α, β, and γ represent coefficients of the forward scattering amplitudes at, respectively, the grid, the plate, and that scattered at the gun (by the perpendicular electric field, for example) but unscattered anywhere thereafter; α_0 is defined by $\alpha = \alpha_0|\psi_p|^2$ (see Ref. [2] or [6] for details).

The term $\sim\cos kL_p$ describes the interference term along with a second harmonic term $\sim\cos 2kL_p$. When the electron energy is swept in the experiment, for a given axial magnetic field and a fixed L_p, we observed a series of maxima and minima. These are shown in Fig. (10a) of Ref. [6] (or Fig. 2 of Ref. [2]). The positions of the interference maxima have been shown in Table 7 of Ref. [6] to fit the relation

$$\Omega L_p = 2\pi l v_\parallel, \quad l = 1,2,3. \tag{3}$$

This clearly corresponds to the interference term cos kLp in expression (2) and demonstrates the existence of the one-dimensional interference phenomenon with wavelength $\lambda = 2\pi v_\parallel/\Omega$.

In another important variation of the above experiment, the grid (which is held grounded throughout the experiment) was systematically moved away from the plate as the latter is held fixed. When the experiment was carried out, as above, by sweeping the electron energy, we found the existence of "beats" modulating the pattern of Fig. (10a) of Ref. [6] or Fig. 2 of Ref. [2]. These beats are shown in Fig. 11 of Ref. [6] (or Fig. 3 of Ref. [2]) for magnetic fields (a) $B = 69$ G, (b) $B = 135$ G and $L_p = 51$ cm, $L_g = 45$ cm. We have claimed that these "beats" are in fact matter wave beats which arise from the interference of transition amplitudes at the plate, originating at the gun and grid. The wave algorithm with the above wavelength for this case $[(L_p - L_g) < L_p]$ yields for the probability density expression (7.17) of Ref. [6] or expression (11) of Ref. [2] which has a term $\sim\cos k(L_p - L_g)\cos kL_p$. This describes a modulation with a "frequency" $(L_p - L_g)$ of the oscillating term $\cos kL_p$ (of frequency L_p) with respect to the sweep of $k = \Omega/v_\parallel$. We use the term "frequency" here with respect to the variation of the wave number k, which is effected through the electron energy. This modulation thus describes "beats" with a frequency $(L_p - L_g)$, which is the difference between the two "frequencies" L_p and L_g in the system. A rearrangement of the terms of the expression (7.17) of Ref. [6] leads to the form

$$|\psi_p|^2 = [2\beta\gamma + 2\alpha_0\gamma(3\beta^2 + 2\gamma^2)]\cos kL_p + 2\alpha_0\beta\gamma^2 \cos 2kL_p - 4\alpha_0\gamma(4\beta^2 + 3\gamma^2)\sin^2\frac{1}{2}k(L_p - L_g)\cos kL_p. \tag{4}$$

This expression gives the beat term in the form of the square of the amplitude modulating factor $\sin\frac{1}{2}k(L_p-L_g)$, multiplying the oscillating term $\cos kL_p$. With respect to the sweep of k (through the sweep of energy as carried out in the experiment) the maxima of the beats would be described by the relation

$$\Omega(L_p - L_g) = 2\pi l v_\parallel, \quad l = 1,2,3 \ldots . \tag{5}$$

When the beats observed in Fig. 11(b) of Ref. [6] are analyzed, they are indeed found to fit the relation (5) with $l=2,3,4,5$. This is demonstrated in Table 8 of Ref. [6]. We consider this as a crucial test of the manifestation of the matter wave nature of the observed phenomenon, and we have therefore stated in our original reference [2] that this agreement with the consequences of the wave algorithm proves unambiguously that the matter wave phenomenon is at play.

We had, in fact, remarked in Ref. [2] that the beats can also occur when two sinusoidally varying particle sources with frequencies L_p and L_g are superposed. But in that case, the beat frequency would be only $\frac{1}{2}(L_p-L_g)$, half the difference (L_p-L_g). These can be referred for short as "particle beats" as against "wave beats" which have a frequency (L_p-L_g). We had thus argued that the observed beats which have been found to have the frequency (L_p-L_g) can be understood only in terms of the wave formalism.

III. CRITIQUE OF THE UNNIKRISHNAN MECHANISM

In an attempt to provide a classical explanation for the observed beats, Unnikrishnan has, we are afraid, tried to argue unsuccessfully (as we shall show in what follows) that the beats observed by us can be explained as "particle beats" and that there is no need to invoke any matter wave formalism in the macrodomain developed in Ref. [5]. Since he identifies the beats observed by us as "particle beats," one should be able to decide the issue by determining whether the observed beat frequency is the difference of the frequencies (L_p-L_g) or half the difference $\left[\frac{1}{2}(L_p-L_g)\right]$. However, we shall show below that he fails to discern the phase relationship across the beat nodes, which decides the frequency of the beats as being (L_p-L_g) or $\frac{1}{2}(L_p-L_g)$ and thereby wrongly (and somewhat peculiarly) asserts that though (in the case of particle beats) the signal at the frequency (using his notation) $\frac{1}{2}(\omega_1+\omega_2)$ is modulated at a frequency $\frac{1}{2}(\omega_1-\omega_2)$ "the beats themselves are at frequency $(\omega_1-\omega_2)$ since the separation between the maxima of the modulated pattern is at time intervals $1/(\omega_1-\omega_2)$."

We shall show graphically the nature of the phase relationship across the beat nodes for the two cases corresponding to "particle beats" and "wave beats," and point out the fallacy in the above assertion by Unnikrishnan. However, to first summarize the actual mechanism proposed by Unnikrishnan, he makes use of a rather fortuitous similarity between the expression for the matter wavelength $\lambda=2\pi v_\parallel/\Omega$, as described above, and the focusing length of a beam of electrons traveling from a source along a magnetic field with a small spread in their pitch angle of injection. It turns out that the focusing length has (fortuitously) the same expression as λ—namely, $l_f=2\pi v_\parallel/\Omega$. It is this similarity which forms the basis of his mechanism. (It must be pointed out right away that there is no counterpart in this fortuitous similarity of the harmonics λ_n of the fundamental wavelength λ which the transition amplitude furnishes. That is, there are no harmonics of the focusing length l_f.)

There are two aspects of his mechanism (model) which need to be examined: (i) The actual feasibility of this mechanism being operative and its consistency and (ii) its ability to explain the detailed structure of our experimental observations. On going through his comments it seems to us that it is seriously wanting on both these counts.

The basic element of his model is to generate two (secondary) electron sources with two different frequencies, so that their superposition leads to the production of "particle beats" in the electron current signal at the plate, modulating a signal with a frequency equal to half the sum of the two frequencies with a beat frequency equal to half the difference of the two frequencies. To achieve that he makes use of the following two stipulations, the first of them being physically justified, but the other not.

(a) The focusing of the electron beam in space at every focusing length $l_f=2\pi v_\parallel/\Omega$, beginning from the source—the electron gun. This is a theoretically justified and an experimentally established property.

(b) The stipulation that the secondary electron emission from any electrode, plate, or grid on which the electron beam falls depend not only on the total number of electrons falling on it but also on the surface beam intensity (number of electrons per unit area) at the electrode.

While (a) is known to be true, (b) is not supported by any systematic published work. The author quotes a private communication in his earlier paper [7] in support of this assumption. This private communication attributed to one of the authors of [2] pertained to some preliminary results that were obtained five years ago and needed, for their confirmation, more systematic and controlled experiments which have not been carried out. More recent work carried out elsewhere [8] in connection with the secondary electron emission in the LHC (Large Hadron Collider) at CERN shows that there is a systematic decrease in the secondary electron yield as the surface is exposed to electron bombardment with a few hundred eV energy and becomes stabilized to a plateau value after a certain total amount of electron dose. These experiments do not support any intensity-dependent secondary electron emission (SEE) but only a stabilized yield. An intensity-dependent secondary electron yield is theoretically possible when the thermal load on the surface is large enough to cause changes in the surface properties. Even then it will require a quick reversal of surface conditions when the load decreases, so as to obtain the kind of variation Unnikrishnan envisages. But with a few nanoampere current in the energy range ≈ 1 keV that we have used in our experiment, it is unlikely that the SEE yield could have any intensity dependence after the surface has been "conditioned" by electron bombardment, which in our experiment has been the case because of several weeks of experimentation with the same plate and grid.

However, even if we assume (b) to be true for the sake of argument we show, in what follows, that the consequences of

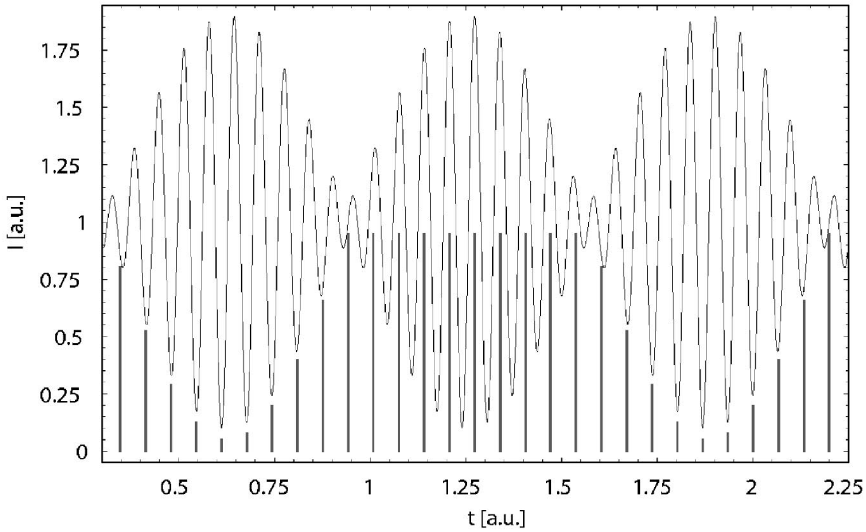

FIG. 1. A plot showing the "particle beats" arising from a superposition of two oscillating particle sources. A scale constructed with the period of the high-frequency oscillations set across the beats clearly shows a phase change of π across consecutive nodes. The numbers in the axes represent the respective quantities in arbitrary units (arb. units).

the model as envisaged by Unnikrishnan are in contradiction with our experimental results in some very essential respects.

Making use of assumption (b), the proposed mechanism associates the focused and defocused states of the beam on the electrodes with varying (SEE)—the focused state being assumed to correspond to enhanced emission—so that the sweeping of the electron energy as carried out in our experiment translates into quasisinusoidal secondary emission from the electrodes—the plate and grid. The periods of the sinusoidal variation with the electron energy sweep are then determined by their distances from the gun. Further, an enhanced secondary electron emission from an electrode would thus correspond to a minimum of the electron current from that electrode.

It may now be pointed out that this mechanism is in contradiction with our experimental results at the very outset. It is stated by the author that when the plate is close to the focus it corresponds to a minimum in the plate current, in accordance with the assumption of enhanced secondary emission in the beam-focused state. A focus close to the plate implies that there are an *integral* number of focusing lengths between the gun and plate. However, this also means that there are an integral number of wavelengths between the gun and plate, since the expressions for the two are fortuitously the same. A look at Table I of Ref. [2] would show, on the other hand, that this (the integral number of wavelengths) corresponds to a maximum of the plate current in our experiment. Since according to the model of Unnikrishnan it should correspond to a minimum in the plate current, the very first step of the model is thus contradicted by our observations. It is somewhat strange that this contradiction at the very first basic level was not noticed by the author. But we may still continue further so as to point out more fallacies in the author's argument and contradictions of his model with our experimental results.

Unnikrishnan exhibits in Fig. 4 of his comment an envelope showing essentially what are "particle beats"—that is, the superposition of the effects of the two oscillating electron sources, resulting in a signal of frequency $(\omega_1+\omega_2)/2$, modulated by a frequency $(\omega_1-\omega_2)/2$, as given by his Eq. (10). Since it looks *similar* to our experimentally observed "beat" pattern, he concludes that it reproduces our experimental pattern. Moreover, as remarked already, he makes a rather astonishing statement that even though the frequency of modulation is $(\omega_1-\omega_2)/2$, " the beats themselves are at a frequency $(\omega_1-\omega_2)$, since the maxima of the modulated pattern is at time intervals $1/(\omega_1-\omega_2)$." It is through this rather strange assertion that he claims to have reproduced beats observed by us in Ref. [2]. But this is fallacious, because the frequency of a signal—beats in the present case—is not determined merely by its appearance or by node-to-node separation as the author seems to have been led by. It refers to all the details of the signal. The beats presented by the author in his Fig. 10 have really a frequency $(\omega_1-\omega_2)/2$ because the high-frequency oscillations in the two consecutive envelopes are out of phase, while they get in phase only in the next envelope—that is, after a period of $4\pi(\omega_1-\omega_2)^{-1}$. This is perfectly consistent with the beat frequency being equal to $(\omega_1-\omega_2)/2$. Our experimentally observed beats, on the other hand, have truly a frequency $(\omega_1-\omega_2)$, because if one looks at the Fig. 4 of Ref. [2] which is a replotted version of Fig. 3 on the inverse velocity scale, we see that the high-frequency oscillations in all the envelopes are in phase, which corresponds to a period of $2\pi(\omega_1-\omega_2)^{-1}$. Moreover, this also implies that the modulating function is positive semidefinite everywhere and thereby corresponds to the amplitude having been squared. Thus what we have observed in our experiments is an unambiguous signature of the wave algorithm and confirms our conclusion that we have observed "wave beats" rather than particle beats in our experiments as against the contention of Unnikrishnan.

We exhibit the above fact graphically which makes it even more transparent. Following Unnikrishnan we plot in Fig. 1 a superposition of two sinusoidally oscillating terms with frequencies $\omega_1=100$ and $\omega_2=90$ in arbitrary units,

$$I = I_0 + a(\cos\,\omega_1 t + \cos\,\omega_2 t)=I_0 + 2a\,\cos\frac{1}{2}(\omega_1 + \omega_2)t\,\cos\frac{1}{2}(\omega_1 - \omega_2)t, \qquad (6)$$

taking, for simplicity, the same amplitude a for the two

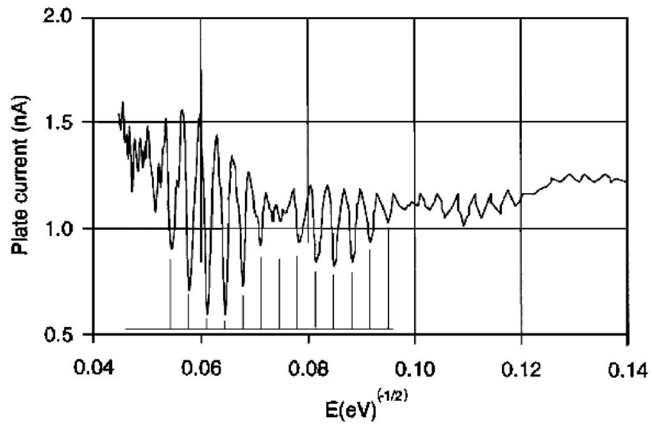

FIG. 2. Plot showing the "wave beats" observed in the experiment (a replotted version on the inverse velocity scale). A scale constructed with the period of the high-frequency oscillations set across the beats clearly shows the oscillations to be in phase across the consecutive beat nodes.

terms. This of course, gives a modulation of the high-frequency oscillation with frequency $(\omega_1+\omega_2)/2=95$, with low frequency $(\omega_1-\omega_2)/2=5$, if ω_1 and ω_2 are close by, as they are. This can be regarded as beats with period $4\pi/(\omega_1-\omega_2)$. (We have continued, in this particular discussion, with the notation of Unnikrishnan, ω and t. In the experimental situation, as we have noted already, $\omega_{1,2}\rightarrow L_{p,g}$ and $t\rightarrow k$.)

To demonstrate graphically that the beat pattern of Fig. 1 does have a period $4\pi/(\omega_1-\omega_2)$, we prepare a scale with divisions, each equal to the period of the high-frequency oscillation, and set it across the oscillatory pattern as shown. It is seen clearly that there is phase change equal to π across any particular node from one envelope to the next, because as we notice, that while the markings on the scale coincide with the maxima on one envelope, they coincide with the minima in the consecutive envelope and coincide with the maxima only in the next envelope. This clearly demonstrates that the beat period is $4\pi/(\omega_1-\omega_2)$, rather than $2\pi/(\omega_1-\omega_2)$, as wrongly claimed by Unnikrishnan, since he failed to notice the phase relationship between consecutive envelopes.

By contrast, if we carry out a similar examination of the beats observed in our experiment represented by Fig. 4 of Ref. [2] (which is a replotted version of Fig. 3 of the same reference on the inverse velocity scale $1/v$, since $k=\Omega/v_\parallel$) and reproduced here in Fig. 2, it is clearly seen that all the marked divisions of the scale constructed with the period of the high-frequency oscillation, $2\pi/L_p$, in the present case coincide with the high-frequency maxima across the node from one envelope to the next. This clearly shows that the beat period is in indeed $2\pi/(L_p-L_g)$ and the beat frequency (L_p-L_g), and this frequency corresponds to the modulating amplitude factor and, thus, to a wave algorithm, where the intensity is obtained as the square of the amplitude.

If we now refer to Eq. (4), where plate current is obtained through application of the wave algorithm, we do note that the beat term has the form $\sin^2\frac{1}{2}k(L_p-L_g)\cos kL_p$, with the modulation factor being $\sin^2\frac{1}{2}k(L_p-L_g)$. Our experimental findings with respect to the beat frequency are thus in agreement with the wave formalism.

We may finally mention that our remark in Ref. [2] relating to the frequency of particle beats being only half the difference (L_p-L_g) was essentially meant to rule out a possible mechanism of the type proposed by Unnikrishnan.

There is, however, another point worthy of attention, though it is of secondary importance comapared to the one discussed above. In our experiments, we have always found that the high-frequency oscillations (with respect to the variation of k) are characterized by the frequency L_p. This is what the expressions for $|\psi_p|^2$ following from our general expression (7.15) of Ref. [6] show, in particular the expression (4) here for the "beats" case. The high-frequency oscillation in the Unnikrishnan model would have the frequency $\frac{1}{2}(L_p+L_g)$ which is not what is observed. For the case of beats observed experimentally discussed above (Fig. 3 of Ref. [2] or its replotted version, Fig. 4), $L_p=51$ cm, $L_g=45$ cm, so that $\frac{1}{2}(L_p+L_g)=53$ cm. But the observed high frequency really corresponds to $L_p=51$ cm rather than to 53 cm. The model proposed by Unnikrishnan thus fails on all crucial counts to explain the experimental results obtained by us, even if we assume the existence of the intensity dependence of the secondary electron yield (a crucial input of the model without which the model cannot even take off) which is not an experimentally established physical fact.

There are other details of the model which one could comment upon, but they are really not relevant as the very premise of the mechanism is wrong and the consequences in contradiction with the observed facts.

There are other points of criticism of our paper which we believe arise from a lack of understanding of the spirit of the formalism which motivated these experiments. It ought to be emphasized, for example, that it is essentially a quantum mechanical problem in the correspondence limit, which happens to reveal rather interesting quantum effects, unfamiliar so far, and no new physical laws are sought to be introduced as the comments repeatedly suggest. The central objects through which these effects manifest are the transition (scattering) amplitudes whose interference leads to the observed effects. These transition amplitudes do not exist *a priori*. They are generated at every scattering object or surface and ought to be regarded as originating from there and then propagating there onwards. They are generated at the grid as the electrons in the initial Landau level N are scattered by the grid wires, whereby they are cicked up or down one or more Landau levels and the corresponding transition amplitudes propagate forward. Similarly the plate is also a scattering surface which generates transition amplitudes which in principle can travel beyond the plate surface where they can interfere with other transition amplitudes reaching there, leading to the detection of interference intensity signals. It needs to be emphasized that before a transition amplitude is generated the electron is propagating as a quantum mechanical plane wave $\exp[i\kappa x]$ with the wave vector along the field $\kappa=mv_\parallel/\hbar$. Therefore, the criticism that a wave with a wavelength much larger than the grid spacing, as that corresponding to a transition amplitude would have, would be reflected completely by the grid is misplaced in view of the clarification of our formalism, as made above.

There is, of course, another feature of our experimental plot—namely, the increasing amplitude of the modulated signal with the increasing electron energy. We have not commented upon this feature in terms of a possible explanation. Our comparison with the theory has so far been limited to the most important aspects—namely, the positioning of the various peaks, envelopes, and their phase relationship. We shall discuss this feature in a future communication shortly. It has to do with the fact that the probability of transitions from the Landau level N would increase with the incident energy striking the grid wires.

Another criticism of our paper refers to the "nonexistence in our data of a signal corresponding to the frequency $L_p + L_g \approx 2L_p$ which our expression in Ref. [2] requires." Such signals are indeed present in our data in the form of higher harmonics of the fundamental whose presence is indicated by the existence of the sharpness of the interference peaks in Fig. 4 of Ref. [2]. In fact Fourier analysis of our data reveals the presence not just of a second harmonic, but of higher ones too. These, in fact, correspond to the existence of the higher harmonics of the fundamental wave vector $k = \Omega / v_{\parallel}$ which are implied in the transition amplitude formalism as presented in the Appendix to our paper [2] and as shown in Eq. (1) here. These correspond to transitions by two, three, etc., Landau level intervals from the initial Landau level.

There is a more general kind of criticism of our work in the form of a disquiet, that our interpretation "implies that there are violations of the Lorentz force law, Maxwell's equations and classical electrodynamics." It implies nothing of the sort. For example, we have said nothing about what the trajectory of an individual electron is between the two scattering episodes. It could still be governed by the Lorentz equation, considering the fact that the Landau interlevel energy which is exchanged between the parallel and perpendicular degrees of freedom of the particle on scattering is only of order 10^{-5} eV. Nor can we see how our interpretation could imply wholesale violation of Maxwell's equations and classical electrodynamics. Perhaps the author got carried away. What our experimental results do represent, however, is a manifestation of quantum effects well into the domain and in the correspondence limit, which has so far been regarded as the domain of classical mechanics. But that does not constitute a violation of classical dynamics. What it does imply is that we need a much greater understanding of the correspondence limit of this system. A brief discussion below attempts to further clarify the situation with regard to the question of the relationship of our results with the correspondence principle.

Further, to see to what extent the Lorentz equation describes the trajectory between two scattering episodes, what is therefore required is more experimentation using such a low-intensity electron source that it corresponds to the passage of one electron at a time. We do not believe that we shall find a violation of the Lorentz equation in describing the trajectory between two scattering episodes. Using low-intensity beams in the limit of "one electron at a time" could also settle the question of whether focusing and defocusing of the beam plays any role, because with one electron at a time the question of focusing and defocusing becomes meaningless.

To be sure, our results do raise several conceptual issues, the foremost being the relationship of this formalism with classical mechanics. What these results do entail is an enlargement of our understanding of charged particle dynamics, not amounting to a denial of any known physical laws. In other words, we need a greater understanding of the interrelationship of the laws in view of such findings. The resolution of the issues will be well served by more specific experimentation.

One may, however, state the issue raised above, as to how is it that contrary to the general understanding and belief (via the correspondence principle), the quantum mechanical system of our problem does not go over into its classical behavior in the limit of large quantum numbers and alternatively, therefore, not in accordance with the Ehrenfest theorem in terms of the expectation values. The issue belongs to the more general question of the relationship between classical and quantum mechanics, a complete discussion of which would warrant a full length paper. We present here only a brief discussion, which we hope will clarify the situation to a sufficient degree.

Actually the passage from quantum to classical mechanics is still a nontrivial issue, and one has to be very careful in defining what one means by this passage. If one means that the classical equations of motion are retrieved from quantum mechanics, then, as is well recognized, the classical motion is followed by the centroid of a highly compact wave packet formed out of a very large number of quasi-plane-waves, centered around a central wave number. Thus while classical motion is thus recovered, the pertinent question from our point of view is to ask whether the quantum character of the system is thus lost as a consequence of this wave packet formation. The answer is obviously *no*, unless it is subject to a decohering scattering process, because the packet is a superposition of coherent waves. Likewise going to the correspondence limit does not imply that the quantum character of the system is lost. The Rydberg atoms are still quantum mechanical objects, though the wave packets formed out of the neighboring states would exhibit classical motion.

What needs to be emphasized in relation to our results is that they do not pertain to any particular state in the correspondence limit or to any wave packet constituted thereof, but rather to some new quantum mechanical objects, which are defined using the neighboring stationary states of the system. These objects are partial transition amplitudes between two neighboring states separated by a quantum number interval $\Delta n = 1, 2, 3, \ldots$, induced by an appropriate perturbation in an experiment which causes a change in one of its quantum numbers (the Landau quantum number in the present experiment). These (partial) transition amplitudes can be defined in any range of quantum numbers including that of the correspondence limit and are themselves wave amplitudes, but with wave numbers Δk which are the differences of the wave numbers of the two states defining them. As a consequence they correspond to a macroscopic matter wavelength given by $\lambda_M = 2\pi / \Delta k$. Our experiments have essentially exhibited the manifestations of this macroscopic matter wavelength. These manifestations are basically of quantum mechanical origin and have no bearing with the classical limit in view of the above discussion.

However, it is worth mentioning that transitions between neighboring states in the corresponding limit do have a classical significance in the context of the old quantum theory, whereby the energy difference ΔE between the two consecutive states, and therefore the energy of the radiation emanating from the energy of transition, corresponds to the classical orbital frequency of the electron in that orbit, while its harmonics come from the transitions between levels differing in their quantum numbers by $\Delta n = 2, 3, \ldots$. The new element here is that the (partial) transition amplitudes are essentially described by a wave function with a momentum which is the difference in the momenta corresponding to the two neighboring states, as we conserve the total energy during the transition. We hope that these remarks clarify the nature of the experimental results and their relationship with quantum mechanics as expressed through the theories of Refs. [4,5].

Our experiments have thus pointed to the existence of a new kind of quantum mechanical wave amplitudes defined by the transition amplitudes between the neighboring states, in particular in the correspondence limit. This led to the extension of the domain of quantum behavior well into the macroscopic domain which is usually regarded as the domain of classical mechanics. We have also shown [9] that similar considerations can be applied to other systems as well, such as atoms and molecules leading to macroscopic and mesoscopic matter waves associated with such systems.

ACKNOWLEDGMENT

The authors would like to thank Dr. Michael Marti for his help with one of the figures and Mr. H. S. Mazumdar for some helpful discussions.

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